LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

LLNL-JRNL-420571

MHD-Stabilization of Axisymmetric Mirror Systems Using Pulsed ECRH

R. F. Post

November 23, 2009

Fusion Science and Technology

MHD-Stabilization of Axisymmetric Mirror Systems Using Pulsed ECRH

Richard F. Post

Lawrence Livermore National Laboratory

Abstract

This paper, part of a continuing study of means for the stabilization of MHD interchange modes in axisymmertric mirror-based plasma confinement systems, is aimed at a preliminary look at a technique that would employ a train of plasma pressure pulses produced by ECRH to accomplish the stabilization. The purpose of using sequentially pulsed ECRH rather than continuous-wave ECRH is to facilitate the localization of the heated-electron plasma pulses in regions of the magnetic field with a strong positive field-line curvature, e. g. in the "expander" region of the mirror magnetic field, outside the outermost mirror, or in other regions of the field with positive field-line curvature. The technique proposed, of the class known as "dynamic stabilization," relies on the time-averaged effect of plasma pressure pulses generated in regions of positive field-line curvature to overcome the destabilizing effect of plasma pressure in regions of negative field-line curvature within the confinement region. As will also be discussed in the paper, the plasma pulses, when produced in regions of the confining having a negative gradient, create transient electric potentials of ambipolar origin, an effect that was studied in 1964 in The PLEIDE experiment in France. These electric fields preserve the localization of the hot-electron plasma pulses for a time determined by ion inertia. It is suggested that it may be possible to use this result of pulsed ECRH not only to help to stabilize the plasma but also to help plug mirror losses in a manner similar to that

employed in the Tandem Mirror.

I) Introduction

As has been long understood in research into the confinement of plasmas by magnetic mirror systems for fusion power, the employment of axisymmetric fields would have many important advantages, both theoretical and practical. From the latter viewpoint, the simplicity and modularity associated with a field generated entirely by circular magnet coils would have many engineering and economic advantages, as compared to the complexity of the magnet coils of toroidal systems such as the tokamak or the stellerator.

From a theoretical standpoint, particularly as it relates to the drivers for instabilities, axisymmetric mirror systems have no parallel currents in their equilibrium state, and the particle drift orbits form a set of closed nested cylindrical surfaces. For this reason the "neo-classical" enhancement of transport across the magnetic field associated with toroidal systems does not occur. These and other considerations have sparked a new interest in axisymmetric mirror-based systems, and with it the re-examination of means for stabilizing the MHD interchange mode. This fundamental instability mode of axisymmetric mirror systems was first analyzed by Rosenbluth and Longmire [1]. Various stabilization means have been proposed and analyzed theoretically, for example in a paper by Ryutov [2]. In that paper there is given, in integral form and in the paraxial limit, a sufficient condition for stabilization of the MHD interchange mode. Ryutov's form of the integral is given below.

$$I = \int_{-L}^{L} a^3 \frac{d^2a}{dz^2}\left[p_{perp} + p_{par} + \rho v^2\right]dz > 0,\ \text{Stable} \qquad (1)$$

Here **a** is the plasma radius, **L** is the half-length of the mirror magnetic fields out to the physical ends of the system (i.e. including the fringing field outside the mirrors and inside the vacuum chamber), and the term in brackets is the value of the total plasma pressure at the position **z.** Note that stability requires contributions to the integral coming from regions where the field-line curvature (second-derivative term) is positive, and particularly favors those regions where **a** is large, such as is the case in the "expander" region outside the outermost mirror. Within an axisymmetric mirror cell there will always be regions with both negative and positive field-line curvature, and in the paraxial limit and with typical mirror-confined plasma pressure distributions the net contribution to the integral will always be negative (subject to minimization by field-shaping, as described by Ryutov[3]). The integral, Equation 1, interpreted to include time-dependent effects, will be employed in the analyses to follow.

II) MHD Stabilization by Pressure-Pulse Trains

The concept being investigated in this paper is the possibility of employing a train of plasma pulses generated by sequentially pulsed ECRH to MHD-stabilize axisymmetric mirror systems. The proposed technique is a variation on the concept of the Kinetic Stabilizer [4] in which ion beams or plasma streams injected into the expander region act to stabilize the mirror- confined plasma.

Our first task is to show that the proposed technique of dynamic stabilization can succeed when applied to the fundamental differential equation describing the exponential growth (unstable) or oscillatory behavior (stable) of an MHD instability, including time-dependent stabilizing terms. This equation takes the form given below:

$$\frac{d^2u}{dt^2} = \left\{\gamma^2 - c\left[ps(t)\right]\right\}u(t) \qquad (2)$$

Here **u(t)** is the amplitude of the MHD oscillation, γ^2 is the MHD growth constant, **ps(t)** is the time-dependent pressure-pulse stabilizing term, and **c** is an amplitude multiplier of that term. The repetitive pressure-pulse terms used in the computation had a form similar to the ones shown in Figure 1 below.

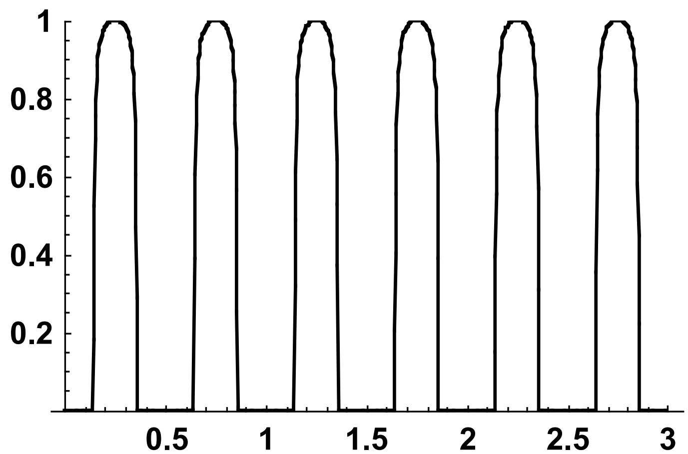
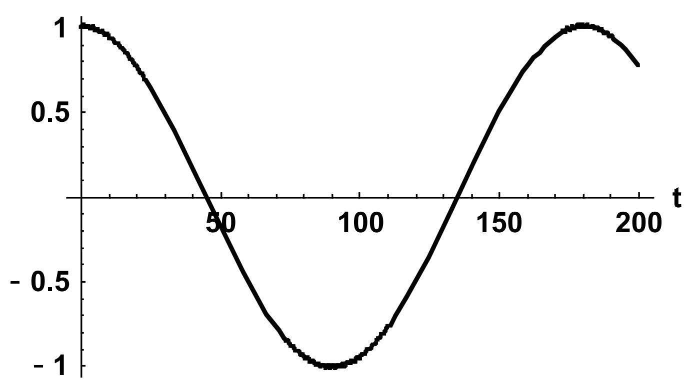


Figure 1: Example of a repetitive stabilizing pressure-pulse train

Equation 2 was programmed for numerical solution and the stable and unstable behavior of the solutions was investigated by varying the parameters. For a case where

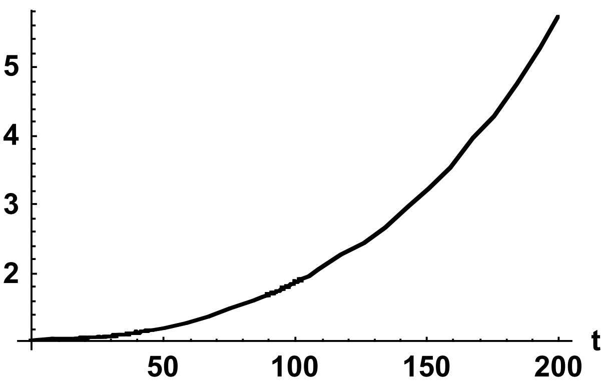


the value of $\gamma^2 = 0.05$ was assumed, corresponding to a characteristic growth time of 4.5 time units, the stabilizing effect of a train of pressure pulses with a period of repetition of 1.5 time units was examined by varying the amplitude multiplier, **c**. As shown in the figures below, stability was achieved when the time-averaged value of the pressure pulses exceeds the MHD growth constant, $\gamma^2 = 0.05$.

$c[ps(t)]_{av} = 0.051$ (Stable)

Figure 2: Solution to MHD equation when the time-averaged stabilizing term exceeded the value of γ^2.

Starting with an initial amplitude of 1.0 the solution to the MHD differential equation is oscillatory (stable). However, as is shown in Figure 3 below, when the time-average of the stabilizing term is less than 0.050, the solution to Equation 2 grows exponentially from an initial amplitude of 1.0.



$c[ps(t)]_{av}$ = 0.049 (Unstable)

Figure 3 : Unstable solution to MHD equation when the time-averaged stabilizing term is less than the MHD growth constant, $\gamma^2 = 0.05$.

Since the stabilization requires only that the repetition period of the pressure pulses be short compared to the MHD growth constant and that their time-averaged value exceeds that given by the growth constant, γ^2, it follows that the same result will be obtained for a pulse train the repetition time of which varies randomly, say over a factor of 3. An example of such a pulse train is shown in Figure 4.

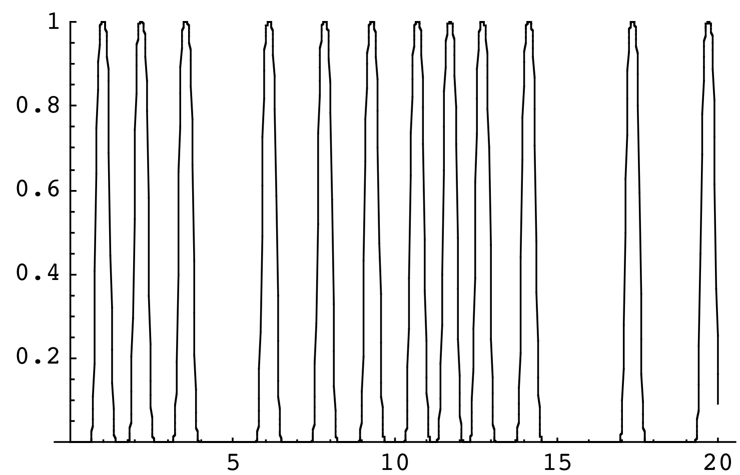


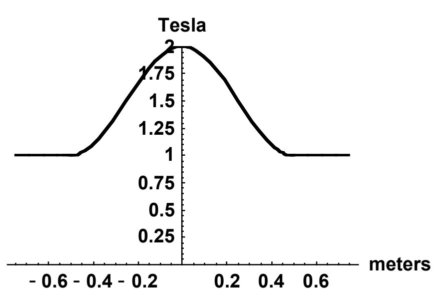
Figure 4: Pulse train with randomly varying repetition period

The possibility of using pulse trains with a randomly varying period could be advantageous in those cases where it became necessary to avoid resonant excitation of some mode the period of which happened to correspond to the periodicity of the stabilizing pulses.

III) Generation of Electric Potentials by Pulsed ECRH

In the PLEIDE experiments carried out in France in the 1960s [5] the ECRH heating of plasma in a simple mirror cell was investigated. With approximately 400 Watts of incident microwave power electric potentials within the plasma in excess of 15 kilovolts were generated, as shown by the spectrum of fast ions emitted from the plasma. The present study utilizes in its analyses the same concept, ECRH power incident on a magnetic gradient, as the process that was present in PLEIDE.

When a pulsed source of ECRH is directed at plasma in a region of the field where there is a negative gradient of magnetic field electric fields and potentials immediately arise within the plasma. The mechanism: The magnetic moment, $\boldsymbol{\mu}$, of the electrons exerts a force of magnitude $\boldsymbol{\mu}\mathbf{gradB}$ directed down the gradient of the magnetic field. This force is then balanced by the ambipolar electric fields that arise as the electrons attempt to move relative to the surrounding ions. Thus, in a simple model of the process, an electrons and an ion will be accelerated together down the magnetic gradient at a rate determined by ion inertia. The result will be the generation of a spreading potential and plasma pressure front that will decrease in amplitude with time. In the context of the present study, this pressure pulse can exert a stabilizing effect if it is localized in a region of the field with positive curvature. The nature of the process involved and the magnitude of the potentials involved can be deduced from a simple physical model of the process. We assume the sudden creation of a cloud of electrons that are mono-energetic in their energy perpendicular to the field lines. We then balance the resulting $\boldsymbol{\mu}\,\mathbf{gradB}$ force by the electric force $\mathbf{eE}$ and calculate the resulting potential distribution. For the simple case of a plasma with an electron energy of 25 keV residing on a mirror peak of parabolic shape as shown in the left-side graph of Figure 5 and the calculated potential is shown in the right-side graph . Here the mass of the ions has been assumed to be "infinite" so as to avoid the necessity of calculating the time-dependent spreading and decay of the potential.



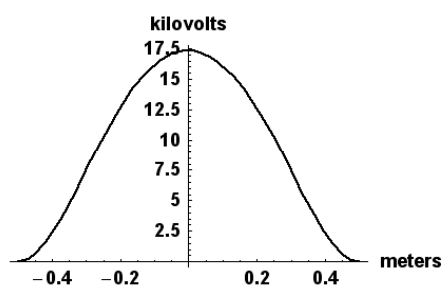


Figure 5: Magnetic field (left plot) and calculated potential (right plot) for 25 keV electrons residing on a mirror field

IV) An Example of a Mirror System Stabilized by Localized Pressure Peaks

As discussed in a previous section, one of the advantages of using pressure pulse trains produced by ECRH is that this technique facilitates the localization of the pressure pulses at regions of the field where the field line curvature is large and positive, thus minimizing the required magnitude of the stabilizing pressure pulses. Further minimization is possible by shaping the flux surfaces of within the mirror cell and by using a sloshing-ion pressure distribution in the confined plasma. In this section we will show the results of calculations made on a mirror cell that incorporates all three of these properties, i.e. stabilizing plasma pressure pulses that are localized in a region of strong positive field line curvature (here, the mirror peaks), mirror flux contours that tend to minimize the regions of negative curvature, and a confined plasma ion pressure that includes sloshing ions (suppressing the Alfvén Ion Cyclotron mode). A plot of the magnetic field strength on axis is shown in Figure 6 (left plot), together with a flux contour (right plot).

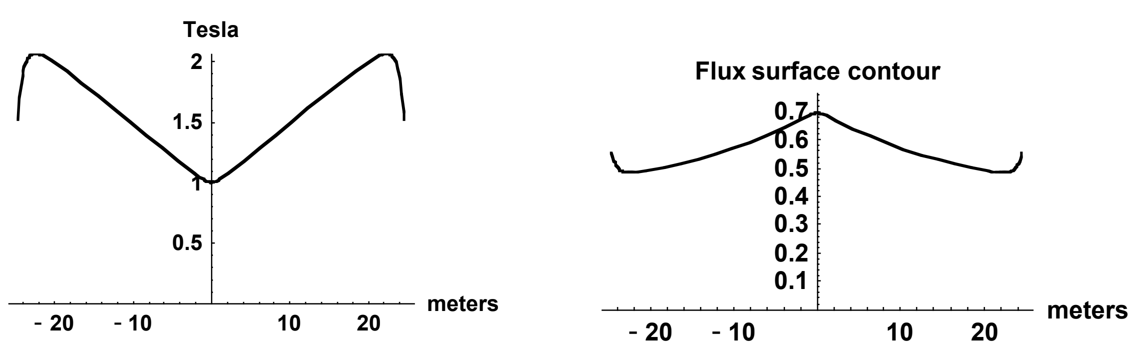


Figure 6: Plots of the magnetic field strength on axis (left plot) and a flux surface contour (right plot) of the example mirror cell

The sloshing ion pressure distribution within the mirror cell, normalized to unity at the peaks, is shown in Figure 7.

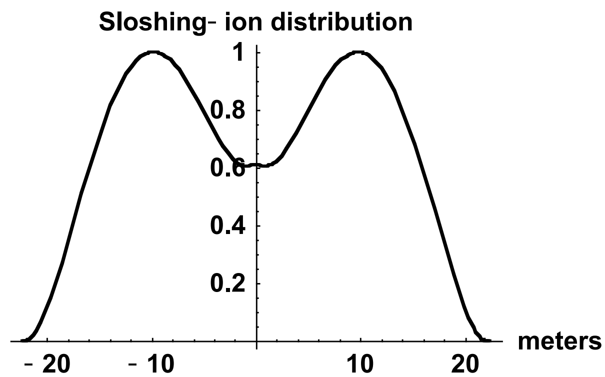


Figure 7: Sloshing-ion pressure distribution in mirror cell of Figure 6

For the distribution shown the computed value of the MHD stability integral, Equation 1, is -0.00311 (unstable). We now add to the total pressure two "stabilizer" pressure peaks centered at the mirror peaks. The maxima of these peaks is 0.09, i.e. 9 percent of the peak pressure of the mirror-contained plasma. Plots of the pressure distribution of the contained plasma (as shown in Figure 7), together with the stabilizer plasma pressure distribution are shown in Figure 8.

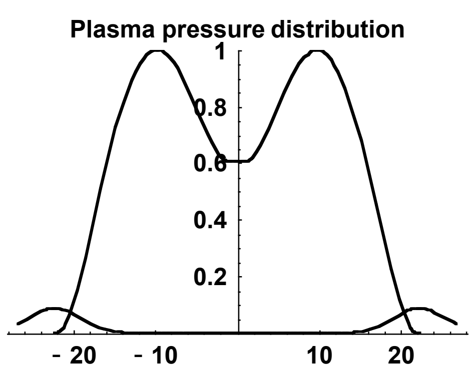


Figure 8: Combined plots of the mirror-contained and "stabilizer" plasma pressures

When we now evaluate the M

IV) Time-Dependent Localization of Pressure Pulses

The stabilizing effect of ECRH pulses will be greatest when they are launched in a region of high positive field-line curvature, for example as shown in Figure 5. However, the ambipolar electric field and potentials generated by the presence of the ECRH pulse (for example, as shown on the right-side plot of Figure 5) will result in spreading out of the pressure pulse on an ion-inertial time scale, the effect of which must be evaluated.

It is possible to estimate the magnitude of this effect from an approximate calculation of the spreading effect, as follows: We consider a thin disc-shaped element of the pressure pulse and then estimate its axial motion by equating the force on each ion in the disc resulting from the calculated local value of the electric field. Because of the gradient in the electric field this process will lead to a broadening of the pressure pulse in a time determined by ion inertia. The net stabilizing effect will then be given by the time-averaged value of the stability integral. The basic equations defining the motion of the disc are those that govern the motion of an ion in the presence of an ambipolar field generated by the **μgradB** force on the electron, coupled to the ion through the electric force, **eE** (Newtons/ion of charge **e**). Thus we have the equation of motion:

$$m_i \frac{d^2 z}{dt^2} = e\mathbf{E} = \mu \nabla B \qquad (3)$$

Here m_i (kg) is the ion mass (here taken to be that of a deuteron), and e is the electronic charge. We will assume here that the electron perpendicular energy is 10 keV. For the magnetic field we will use a parabolic-shaped mirror field similar to that shown in Figure 5, but scaled to dimensions more appropriate to a full-scale mirror system.

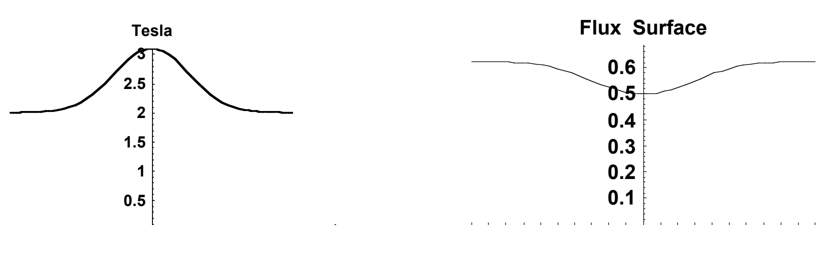


Figure 9 (right plot) shows the spatial variation of the magnetic field; the left plot shows the shape of a flux surface of that magnetic field.

Figure 9: Variation of magnetic field (left plot) and a flux surface (right plot)

In the calculations the initial form of the stabilizing pressure pulse is approximated by a rectangle of unity height and a width that is small compared to the width of the mirror field. As time proceeds this rectangle grows in width and decreases in height (reflecting the conservation of the number of particles in the pulse). From solving Equation 3 we may then plot the shape of the pressure pulse at selected times, as shown in the three plots shown in Figure 10 below. The time is measured in dimensionless units as defined through the initial value of the acceleration at the outer edge of the pressure pulse. In these plots a unit of time is equal to 6.6 microseconds.

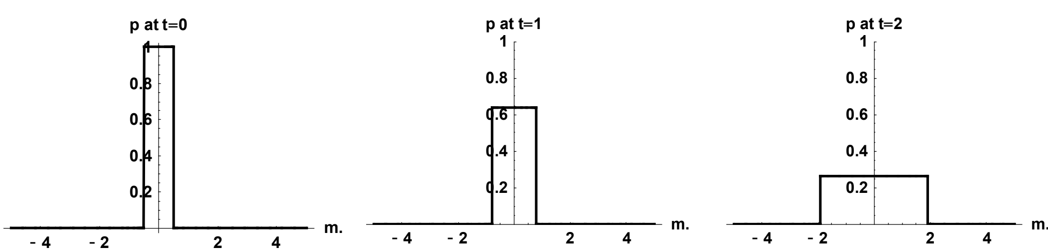


Figure 10: Calculated evolution of the pressure pulse with time

Using the calculated evolution and broadening of the pressure pulse with time the MHD stability integral was calculated as a function of time. As can be seen from the results the decrease in peak amplitude of the pressure pulse is initially largely compensated for by the increased length of the pressure pulse.

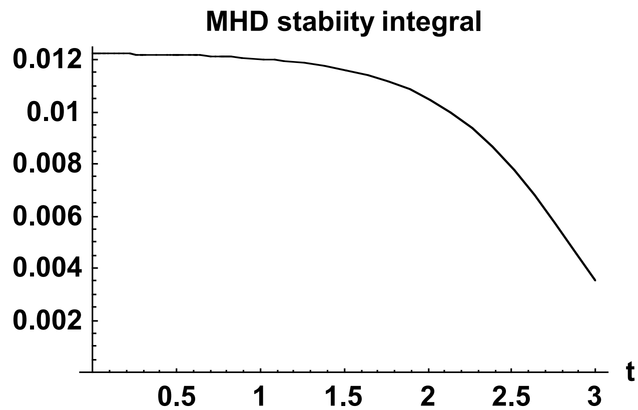
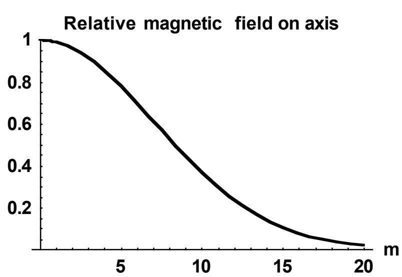


Figure 11: Calculated variation of the MHD stability integral with time.

The peak (t = 0) value of the MHD stability integral is 0.0122, while the time – averaged value (between t = 0 and t = 3) is 0.0104. thus if the pulse is repeated, for example, every 5 time units the overall time-averaged value will be of the order of 60 percent of the peak value for this example case. Whether or not this level of stabilization can be achieved in a practical case will depend on the numbers: size of the system and the details of the magnetic field; availability of high-power pulsed microwave sources, etc.

V) Stabilization by Pressure Pulses in the Expander

The use of ECRH-produced pulse trains to produce stabilizer plasmas in the expander region of an axisymmetric mirror or tandem mirror system, resembling the techniques proposed in the Kinetic Stabilizer concept [4] would have several technical advantages. First, for optimized expander configutations the time-averaged pressures required could be orders of magnitude smaller than those required in other regions of the field. Second, the r.f. frequencies are lower, widening the availability of suitable r.f. generators and at the same time facilitating the achievement of azimuthally symmetric radiation patterns from the launching antennas. In this section we will present some approximate calculations of the time-averaged stabilization occurring when a train of ECRH-generated plasma pulses is launched in a simple expander field. For the expander field we will, to facilitate computation, assume a Gaussian decrease of the field from its throat at the mirror, recognizing that this configuration is less than optimum. The variation of the magnetic field on the axis together with a flux contour of such an expander are shown in Figure 12.



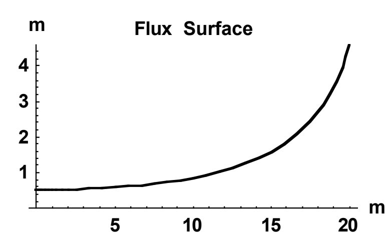


Figure 12: Relative magnetic field on axis (left plot) and a flux surface (right plot) for the Gaussian expander.

A “3-D” plot of the bounding flux surface of the Gaussian expander is shown in Figure 13.

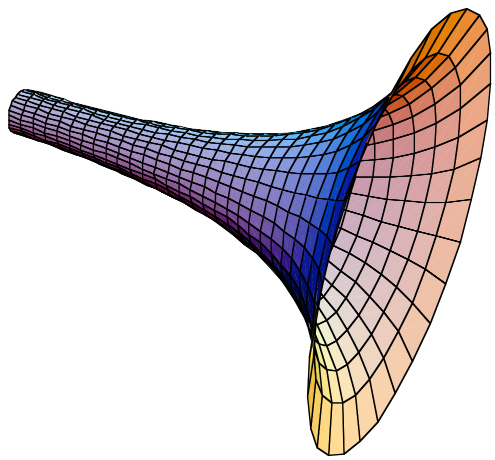


Figure 13: 3-D plot of bounding flux surface of the Gaussian expander

We now consider the time history of a zeroth-order rpresentation of the ECRH-generated plasma pressure pulse, namely, a disc-shaped rectangular-shaped pressure pulse. These pulses are launched down the Gaussian expander from a chosen position located between the throat of the expander and its end. The pulses will be accelerated down the field gradient, expanding radially as they move along the flux surfaces. The time-averaged value of the contribution that these pulses make to the MHD stability integral will then be used to evaluate their ability to stabilize an unstable mirror cell, such as the one shown in Figure 6. Figure 14 is a collage showing the position and amplitude of the pressure pulse at three successive times as it moves outward in the expander. The decrease in amplitude with time arises from the radial expansion of the pulse associated with the outward expansion of the flux surface.

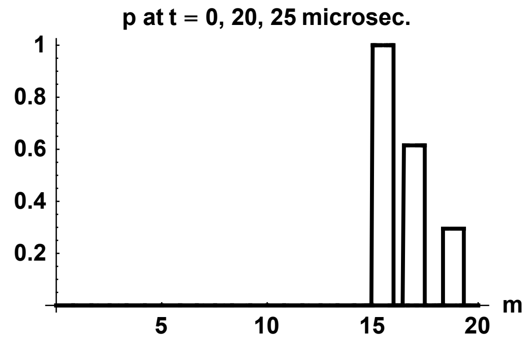


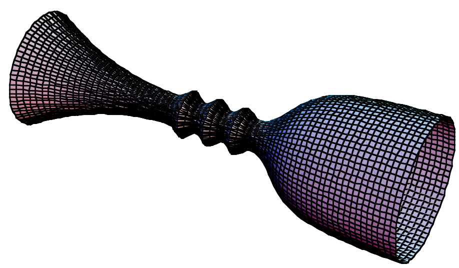
Figure 14: Plots of the position and amplitude of the pressure pulse at its launching , t = 0, and at t = 20 and t = 25 microseconds

When we now calculate the value of the MHD stability integral as a function of time and perform a time average we find the value for the stabilizing effect of a pulse train to be equal to +0.183. Comparing this value to the value of the MHD integral for the mirror cell shown in Figure 6, namely -0.00311, a factor of 60 larger. This result reflects the fact that the pressure in the stabilizer can be much lower than that of the confined plasma and still be effective. However, a pressure ratio of a factor of 60 would only allow the stabilization of a low beta plasma in the mirror cell. In other work [6] it has been shown that there are expander designs that are much more effective than the simple Gaussian expander. These designs lead to mirror-cell/expander pressure ratios that are more than an order of magnitude larger than the value calculated here. Checks of the results using the FLORA [7] MHD stability code also showed that optimized expanders can be used to stabilize mirror-confined plasmas with beta values as high as 40 percent.

In the implementation of any stabilization technique that involves plasma in the expander region there is a concern that insufficient electrical communication between the stabilizer plasma and the main plasma might allow the growth of so-called "trapped particle" modes [8]. In the case that we are considering here, involving low density plasma, either originating from the effluent particles that come from the confinement zone or from a plasma source within the expander, this concern must be addressed. One way that this problem might be alleviated is to employ the pulsed ECRH heating technique not only in the region near the end of the expander as in the example that has been given here, but also in regions close to the throat of the expander. In this way one would create an outgoing flux of plasma pressure pulses that intuitively would modify the boundary conditions of incipient trapped-particle modes and thereby possibly suppress their growth. To validate such speculations would require an extension of the theory to include time-dependent boundary conditions.

VI) Multiple-Mirror End Plugs and Pulsed ECRH-Produced Plasmas

There is another role that might be played by ECRH-produced pulse trains. This role is in connection with the use of multiple mirror cells to plug end leaks. In a study of this technique Post and Li [9] analyzed this problem and showed that the plugging ability of end cells could be enhanced if means could be found to bias the loss probabilities of the end cells in favor of inward losses. In this case it was found that the confinement time of the central cell increased exponentially with the number of cells, rather than linearly, as it does when the outward and inward loss probabilities are the same. Intuitively one might expect that the potential peaks created by pulsed ECRH power ncident on the mirror peaks of a multiple-mirror end-plugging system might provide a means to accomplish this end. However, one cannot use the Post/Li theory to calculate the effect since that theory does not include time-dependent effects. There is, however, one new aspect of the pulsed-ECRH technique that, when employed in a multiple-mirror end-plugging system, would have some unique and possibly very valuable qualities. First, as was described in Section III, when incident on a mirror peak pulsed ECRH results in the creation of potential peaks that could provide a potential-plugging action



similar to the one that is operative in tandem mirror systems. A schematic picture of such a multiple mirror system is shown in Figure 15.

Figure 15: 3-D schematic view of an axisymmetric multiple-mirror end plug and central cell

Conceptually one could therefore visualize an effective tandem-mirror-like potential plugging based on a series of transient potential peaks, properly interleaved in timing and pulse length, caused by pulsed ECRH incident on the mirror peaks of a multiple-mirror plugging system. Here the parallel pressure of the plugged plasma would be balanced by the **μgradB** force exerted on electrons located on the inner faces of the mirror flux surfaces, as transmitted via the ambipolar electric fields. This ECRH-based means of plugging could be an attractive alternative to the conventional tandem-mirror end plugs, with their requirement for high plasma density and/or thermal barriers.

In addition to its possible role in plugging, the MHD stabilization properties of pulsed ECRH plasma (in the form illustrated in Section IV) could be taken advantage of to MHD-stabilize an axisymmetric multiple-mirror end plugging system and its central cell. It remains for future study to determine the viability of these concepts in practical situations.

VII) Conclusions

This paper, a first step into largely uncharted territory, explores a possible new method for the stabilization of axisymmetric mirror systems. Its simple models and approximate calculations are intended to scope out the concept of stabilization by the use of repetitive pulse ECRH heating of selected regions of the magnetic fields of such mirror systems. The purpose of the use of repetively pulsed ECRH is to facilitate the localization of the electron pressure pulses generated by the ECRH. In some cases the pulsed technique offers advantages not possessed by systems based on continuous-wave ECRH. Clearly, to establish the viability and to elucidate the practical problems of implementing the technique would require further theoretical and computational study, backed up by University-scale experiments. It is hoped that this paper may lead to such studies.

Acknowledgments: The author wishes to thank Dmitri Ryutov for helpful discussions. This work was performed under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

References:

[1] C. L. Longmire, "Elementary Plasma Physics," Interscience Pub. (1963).

[2] D. D, Ryutov, "Axisymmetric MHD-Stable Mirrors," Proc. of Course and Workshop,Varenna Italy, Vol II,791 (1987).

[3] D. D. Ryutov, Private communication.

[4] R. F. Post, "The Kinetic Stabilizer: A Simpler Tandem Mirror Configuration?" Transactions of Fusion Technology, Vol. **39**, 25 (2001)

[5] R. Bardet. T. Consoli, R. Geller, Phys. Ltrs **10**, (1964)

[6] R. F. Post, T. K. Fowler, R. Bulmer, J. Byers, D. Hua, and L. Tung, "Axisymmetric Tandem Mirrors, Stabilization and Confinement Studies," Transactions of Fusion Technology, **47**, 49 (2005)

[7] B. I. Cohen, R. P. Freis, W. A. Newcomb, Phys. Fluids **29**,1558 (1986)

[8] M. N. Rosenbluth, Phys. Rev. Lett. **46**, 1525 (1981)

[9] R. F. Post, X. Z. Li, Nuclear Fusion **21**, 135 (1981)